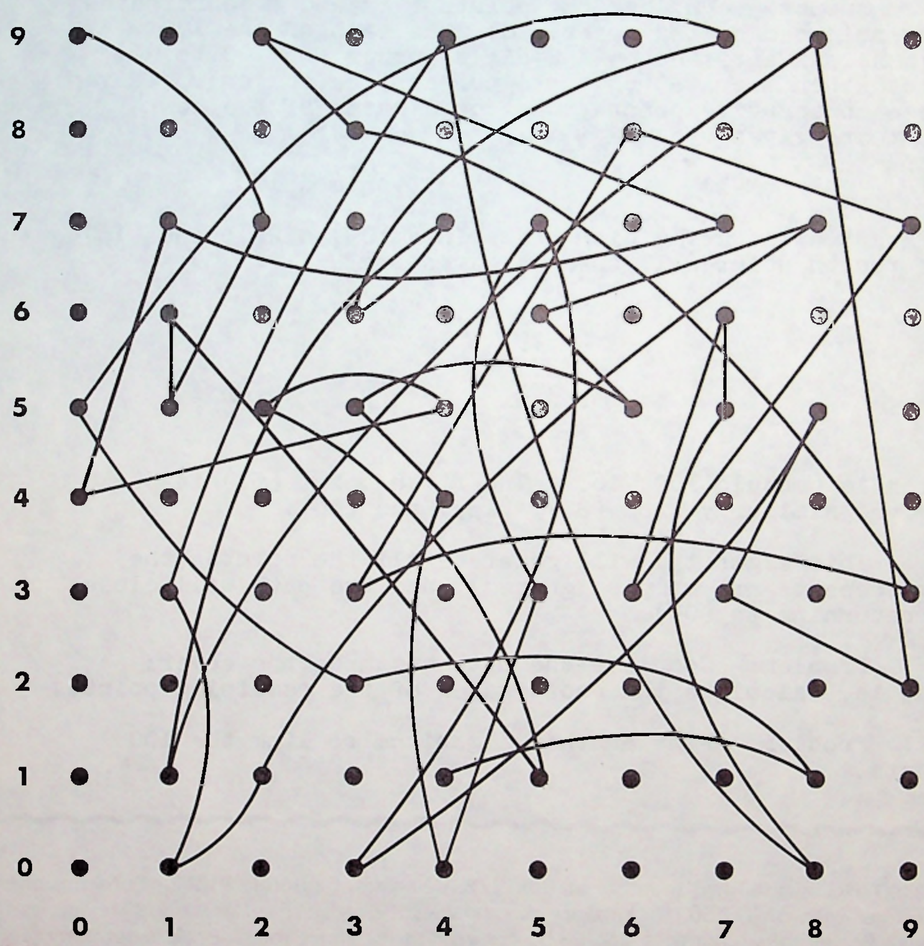
$2.50

54

Popular Computing

September 1977 Volume 5 Number 9



A Unique Path

A Unique Path

The number of paths that can be drawn connecting 100 points in a continuous string is

$$100! = 9.332621544 \times 10^{157}.$$

We would like to reduce that number to unity, by finding an algorithm to link one point to the next. This, it turns out, is not easy.

Suppose we number the points by x and y coordinates (as indicated on the cover), so that each of the 100 points is identified by a 2-digit number from 00 to 99. We can then operate on those numbers, as follows. Given three consecutive points, the coordinates of the next point are given by:

$$N = (N-1)^3 + (N-2)^2 + (N-3) + 3 \bmod 100$$

We start arbitrarily with the points (09), (27), and (15). The fourth point will then be given by:

$$\begin{array}{r} 09 \\ 729 \\ 3375 \\ \underline{3} \\ 4116 \end{array}$$

which is (modulo 100) 16. The fifth point (51) is derived similarly from (27), (15), and (16).

This algorithm will generate all the points (the many repeats are simply ignored), and the path will close by returning to (09).

Problem: Complete the path begun on the cover; that is, calculate the coordinates of the remaining points.

Problem: Find another algorithm to link the 100 points.

PROBLEM 185

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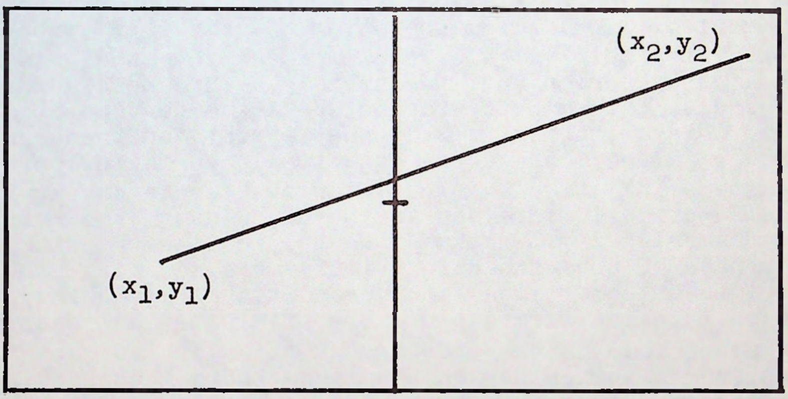
Exploring Random Behavior--1

A point is chosen at random in each of two adjacent squares:



The line connecting these points has an equal probability of crossing the line of division above or below the midpoint. Repeated trials, in fact, will show a distribution much like this:

Number of trials	Number above	Number below
80	43	37
140	71	69
230	118	112
260	132	128

If the dividing line is taken 2/3 of the way up the line of division between the squares, what sort of distribution can be expected? 3/4 of the way up?

Vanderburgh on Calculators

The article "Schwartz on Calculators" in issue 48 brought forth the following from Richard C. Vanderburgh, who is the publisher and editor of 52 Notes:

I think Mort misses the point when he argues that some of the same effects of program modifiability can be produced by indirect addressing maneuverings. Given unlimited data and program memory, and no concern about execution time, one ought to be able to do just about anything with either the SR-52 or HP-67 (or the HP-65, the HP-25, or the SR-56, for that matter). If you look at the whole spectrum of machines from the 4-banger hand-helds to multi-million dollar operating systems from a configuration flexibility point of view, I would say that there is still a useful criterion (not necessarily a threshhold) that can be used to determine how computer-like a particular machine is. This criterion is: To what extent can the user, via software alone, change the functional configuration of the machine being judged. The 4-banger user is stuck with the manufactured firmware, as is the HP-80 user; the primitive programmables limit the user to the in-line ordering of sequences of predefined functions; more advanced programmables provide for conditional and unconditional branching; features locked into the firmware of more primitive machines, but not accessible to the user. At this level of sophistication, the machines now on the market have many handy built-in functions from trig push-buttons to smart card readers, but these are not modifiable by the user, and hence don't qualify as computer-like features by my criterion. At the top end of computer-like features, I would include micro-programability. For example, if I could change bit allocations between mantissa and exponent in a floating point machine without having to make any hardware changes, this would be a very computer-like feature. It doesn't matter whether this machine is a home-brew desk-top, an HP-67X, an SR-52X, or an IBM 370 operating system. It's the degree to which the user can change the functional configuration that counts.

All this leads to how I think the SR-52 and HP-67 should be compared vis-a-vis their computer-like qualities. While block transfers of data from primary to secondary registers, automatic drive motor turn-on when a magnetic card is inserted, and flags that reset themselves following test are features that can be helpful to the HP-67 user under certain circumstances; they limit what he might want

to do in others. On the other hand, the SR-52 user can gain access individually to any one of 60 storage registers, use 28 of them for either program or data storage, 10 of them as either arithmetic stack elements or for data storage, and all of them for indirect addressing and register arithmetic. He can get magnetic cards to be read or written upon under program control; and he can use any one of the flags in the same way as any other. Perhaps the most computer-like SR-52 feature of all is the user ability to create "pseudos"--a capability closely related to micro-programming. While I will grant that user experiments so far haven't yet revealed many "practical" uses for pseudos, "usefulness," like beauty, is in the eye of the beholder, as Knuth, Hamming, and Gruenberger have so ably demonstrated in their toy-program dialogues. But I would be derelict if I didn't note that there is one HP-65/67 feature that is even more computer-like than you'll find in most computers (at the assembly language level) and that is the user ability to choose to have other than a branch occur following a met-test decision point.

To sum up, by my criterion, for machine A to be more computer-like than machine B does not guarantee that A is more useful; only that there are more, or more significant ways for the user to configure it functionally. Machine B may do more hard-wired things than A, but the user can't change these things, or how they are accomplished. By this criterion, I think the SR-52 is significantly more computer-like than the HP-67.

But computer-like qualities aside for the moment... what applications programs can be fit on one HP-67 card that can't be fit on one SR-52 card? I doubt that a 5 x 5 determinant and inverse program can be made to fit on one HP-67 card, and you can't even begin to write an assembler or interpretive computer simulator without run-time access to program registers. Or how about a 32 element difference table?

There are, of course, categories of applications where the HP-67 outperforms the SR-52, notably statistics and certain types of games. But in defense of the SR-52, I wish to clear up some inaccuracies and misleading statements in Mort's article:

1. Parentheses operations do not allow data to slip away (unless the programmer does not understand their use).

2. The implication that HP machines save operands following arithmetic or algebraic operations is only partially true; last x is saved, but not last y. The SR-52 preserves last y in register 99 in the sequence:

y - *EXC 99 = .

3. The ten SR-52 stack registers (60-69) can serve as fully functional data registers when individually addressed. Data pushed into them during stack buildup are reformatted and attached to operators, and hence are not the original data. The HP x, y, z, and t registers cannot be individually addressed, used to perform register arithmetic, or used for indirect addressing.

4. Compared with the SR-52, the HP-67 indirect addressing capability is primitive: only one register for indirect addressing versus 60 for the SR-52. HP-67 program registers are not addressable at all as registers (only individual program steps can be addressed).

I hope I have succeeded in contributing something worthwhile to the growing discussion concerning calculator/computer qualities and characteristics. For the record, I was weaned on HP machines, and still prefer them for manual use. But when the HP-67 first appeared, it didn't seem to me to have significantly more to offer the serious programmer than was already available in the SR-52... and I still feel the same way.

Pi As A Root

The equation:

$$x^5 + 17x^4 + 103x^3 + 239x^2 + 40x - 7640 = 0$$

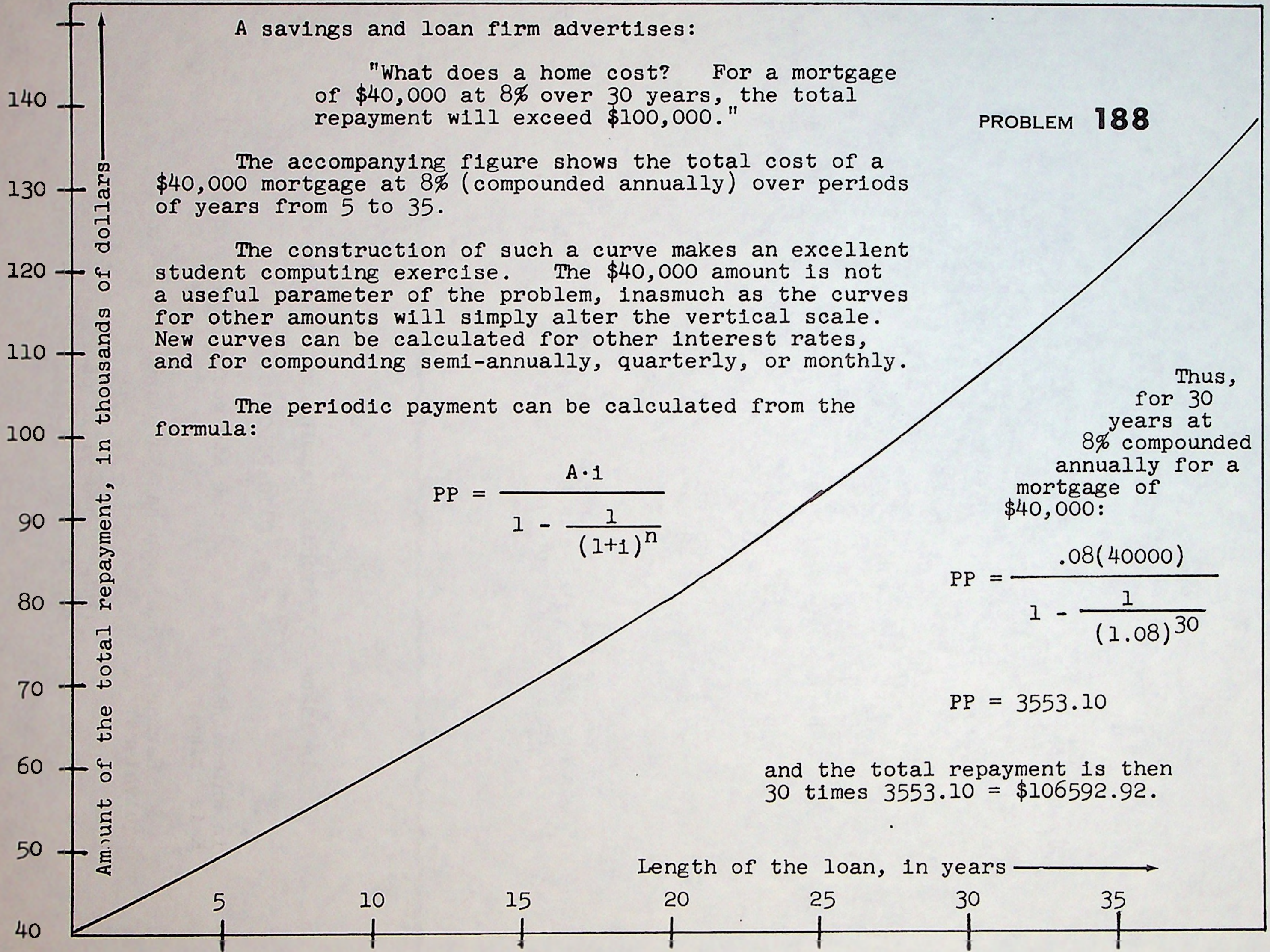
has a root, x = 3.141576...

How close could we come to a root $x = \pi$ with:

1) A polynomial of degree 5 or less;

2) Having integral coefficients each less than 10,000 in absolute value?

PROBLEM 187



PROBLEM **188**

A savings and loan firm advertises:

> "What does a home cost? For a mortgage of \$40,000 at 8% over 30 years, the total repayment will exceed \$100,000."

The accompanying figure shows the total cost of a \$40,000 mortgage at 8% (compounded annually) over periods of years from 5 to 35.

The construction of such a curve makes an excellent student computing exercise. The \$40,000 amount is not a useful parameter of the problem, inasmuch as the curves for other amounts will simply alter the vertical scale. New curves can be calculated for other interest rates, and for compounding semi-annually, quarterly, or monthly.

The periodic payment can be calculated from the formula:

$$PP = \frac{A \cdot i}{1 - \frac{1}{(1+i)^n}}$$

Thus, for 30 years at 8% compounded annually for a mortgage of \$40,000:

$$PP = \frac{.08(40000)}{1 - \frac{1}{(1.08)^{30}}}$$

$$PP = 3553.10$$

and the total repayment is then 30 times 3553.10 = \$106592.92.

Problem Solution

The Animation Problem, Number 159 (PC47-1) presented 16 points on a 100 x 100 grid. At the time of a move, each point moves toward its next numbered point, one-seventh of the distance. After repeated moves, the points should converge; the Problem was to find the area of that convergence.

The accompanying Figures show the results as plotted by Dorothy Cady, of the Computing Center at California State University, Northridge. Mrs. Cady carried the procedure through 500 moves. The coordinates of the 16 points after the 500th move are as follows:

1.	55.48 48.65	2.	55.47 48.59	3.	55.43 48.55	4.	55.37 48.53
5.	55.31 48.53	6.	55.24 48.56	7.	55.19 48.60	8.	55.15 48.66
9.	55.14 48.72	10.	55.16 48.78	11.	55.20 48.82	12.	55.25 48.85
13.	55.32 48.84	14.	55.38 48.82	15.	55.44 48.77	16.	55.47 48.71

The average of the x- and y-coordinates after the 500th move is 55.3125 and 48.6875 respectively, which is the average of the coordinates of the initial positions.

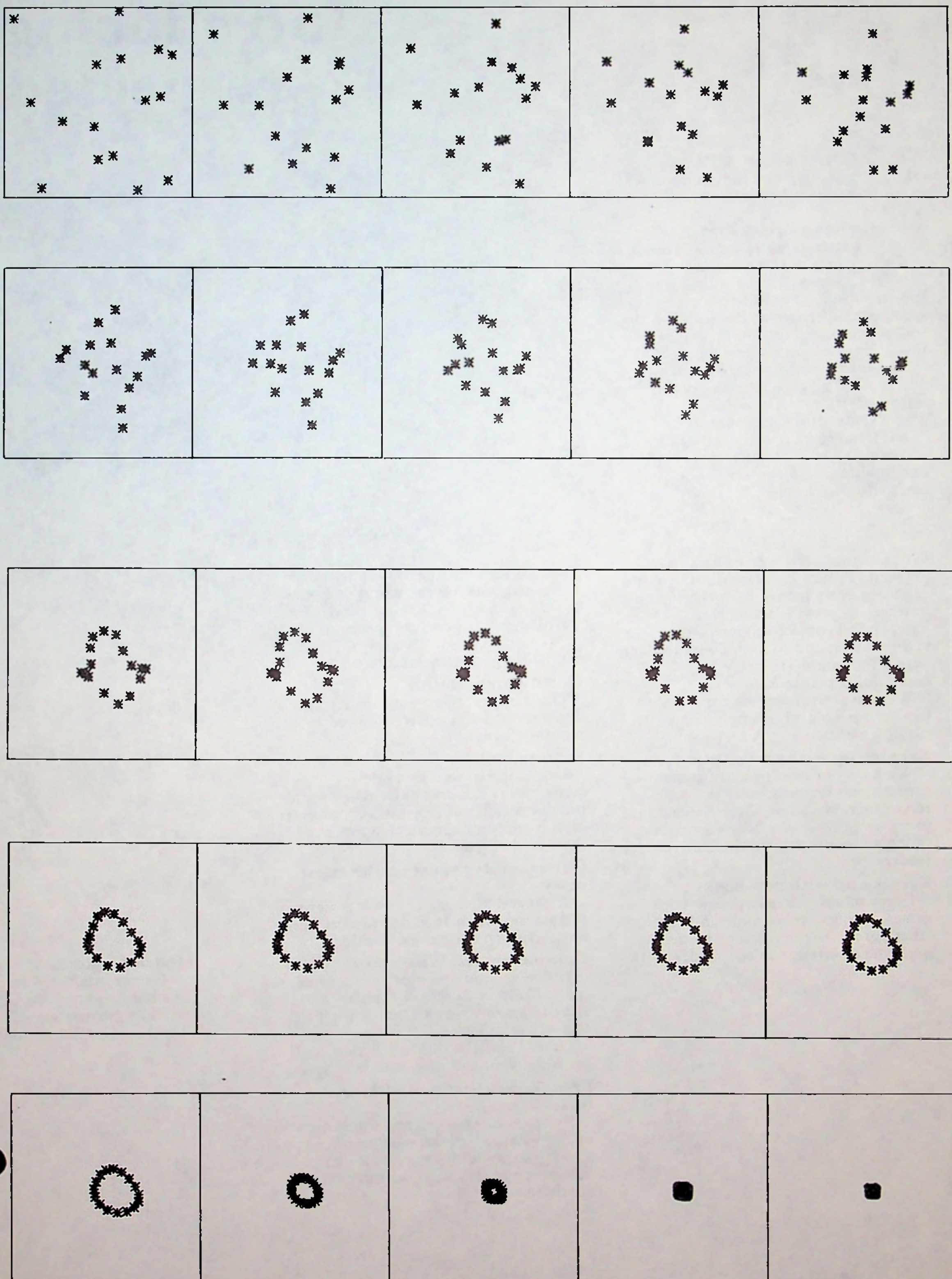
In issue 50, Richard Hamming offered the number

G = .0110101000101...

in which there is a 1 in the Kth position if and only if K is prime.

Herman P. Robinson has furnished the decimal equivalent:

G = .41468 25098 51111 66024 81096 22154
30770 83657 74238 13791 69779 ...



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PC54-12

Marvin's Problem

In the Journal of Recreational Mathematics, 1976-77, No. 3, page 213, Les Marvin has the following problem:

1	1	2	3	5	8	13		
1	2	3	5	8	13	21		
1	3	4	7	11	18	29	47	
1	4	5	9	14	23	37	60	
1	5	6	11	17	28	45	73	
1	6	7	13	20	33	53	86	
1	7	8	15	23	38	61	99	160
1	8	9	17	26	43	69	112	181
1	9	10	19	29	48	77	125	202 ...

For each row, the two numbers to the left of the vertical line are starting values for Fibonacci-type sequences (i.e., $a_n = a_{n-1} + a_{n-2}$). The circled numbers form a sequence of their own. The generation of this sequence makes a splendid coding exercise for a beginning class. The accompanying flowchart indicates one direct solution.

Marvin's problem suggests the formation of another transcendental number:

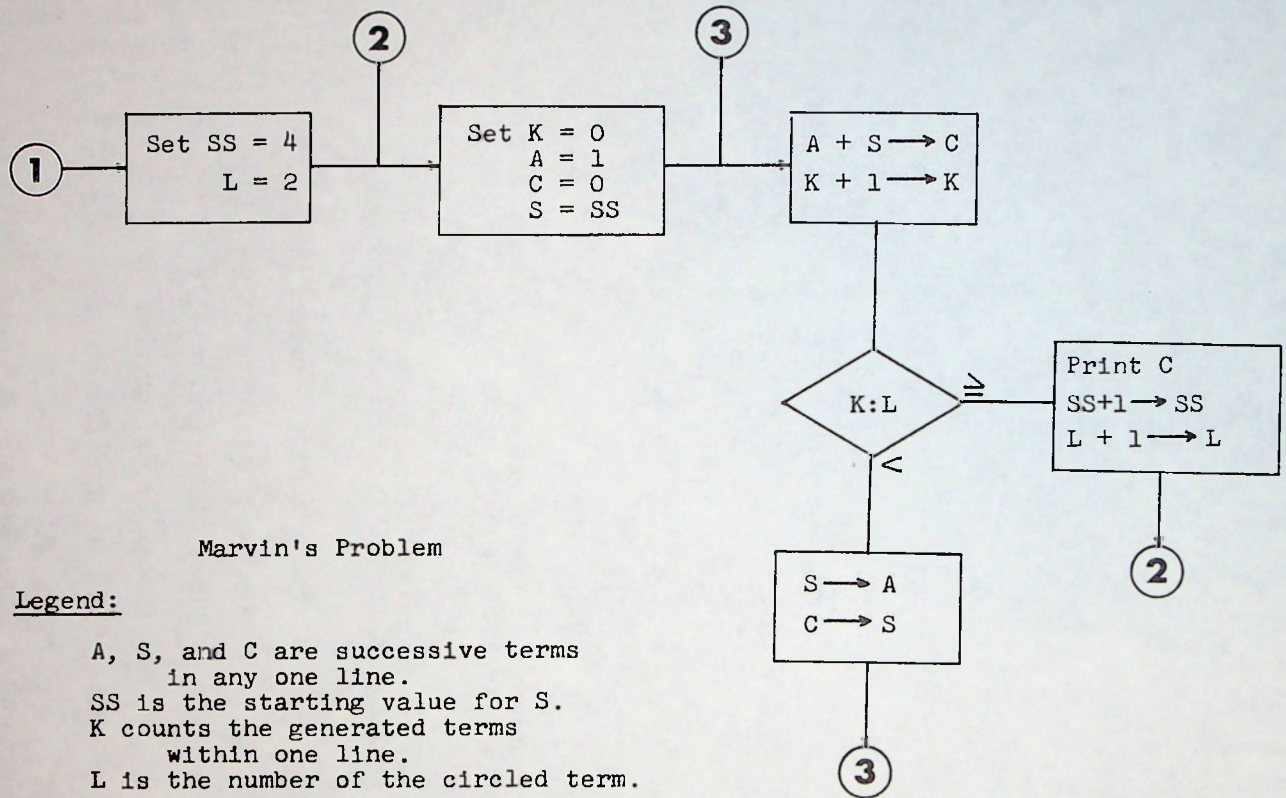
K									
1	1	0							
2	1	4	1						
3	1	7	3	2					
4	2	0	0	0	0				
5	2	2	3	6	0	6			
6	2	4	4	9	4	8	9		
7	2	6	4	5	7	5	1	3	
8	2	8	2	8	4	2	7	1	2

in which the Kth decimal place in the square root of K is selected, producing the number

$$GG = .012069320157946...$$

The calculation of GG to 100 places or more is a non-trivial computing problem.

PROBLEM 189



Marvin's Problem

Legend:

A, S, and C are successive terms
in any one line.
SS is the starting value for S.
K counts the generated terms
within one line.
L is the number of the circled term.

In issue 19 (October 1974), Problem 63 was the following. The lattice lines for the odd primes are drawn in both directions. At the limits set by the 45 degree line, the ratio of the area enclosed by squares to the total area goes as follows:

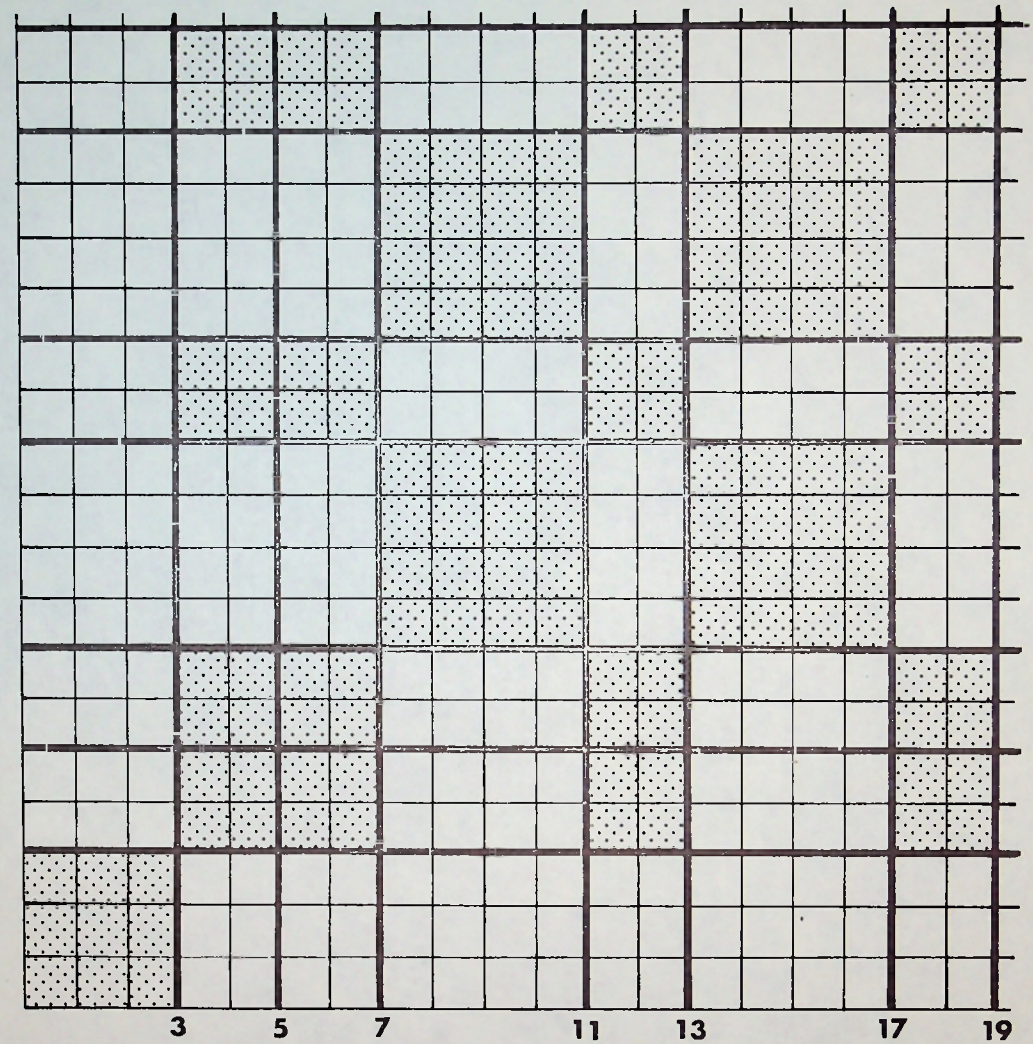
limit on the 45 degree line	total area	area enclosed by squares	ratio
5	25	13	.520000
7	49	25	.510204
11	121	41	.338843
13	169	61	.360947
17	289	109	.377163
19	361	137	.379501
23	529	217	.410208
29	841	253	.300832
31	961	289	.300728
37	1369	397	.289993

The problem that was posed was: what happens to the ratio as more primes are considered?

The method of calculation is simple. Suppose we have just calculated the ratio for P = 103 and we have in storage:

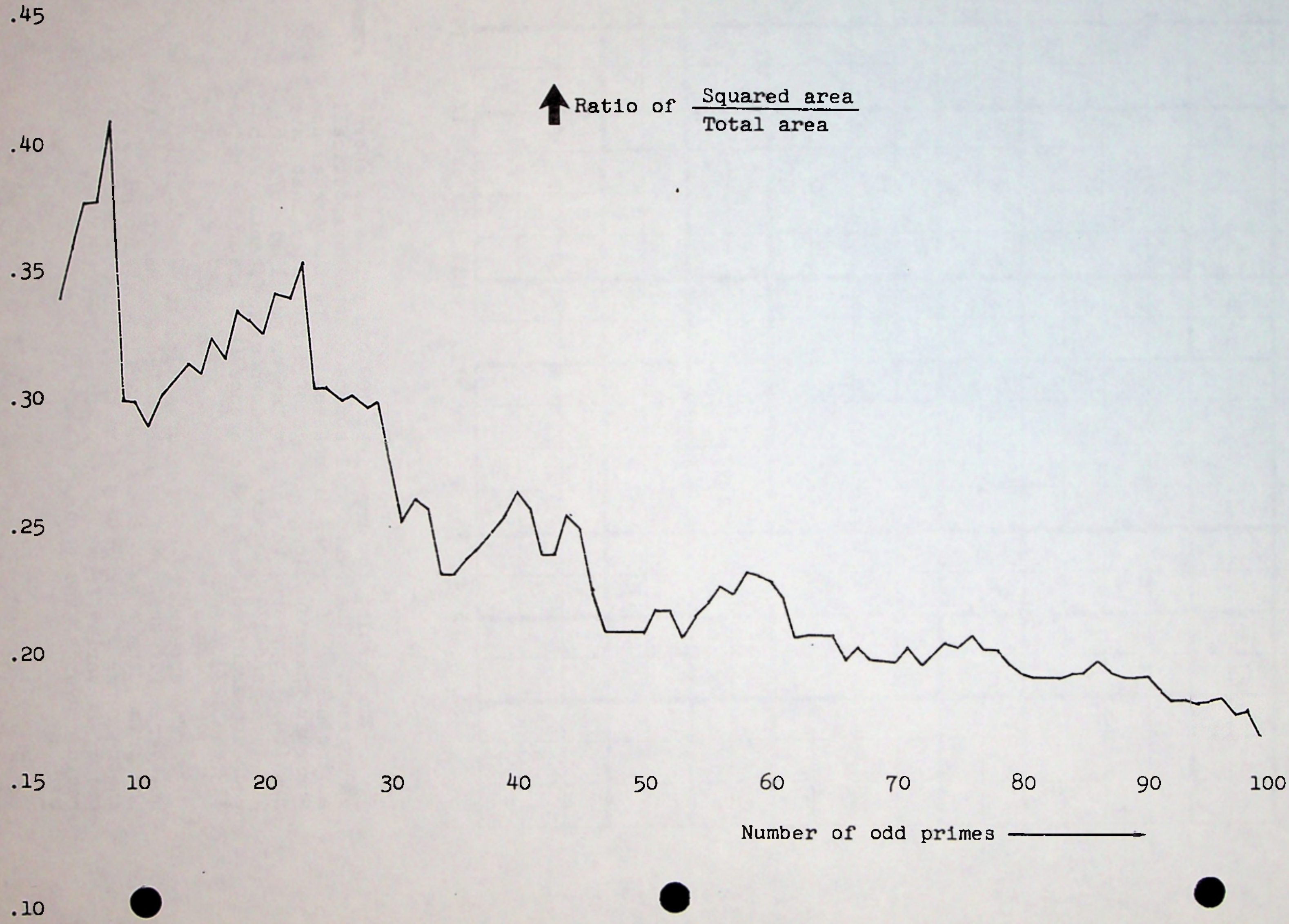
Total "squared" area	3185
Number of differences of 2	9
Number of differences of 4	8
Number of differences of 6	7
Number of differences of 8	1

Find the next prime (107) and the difference with the previous prime (4). Take the number of previous differences of 4 (8), double it and add 1 (17) and multiply by the size of the squares newly formed (16). This gives 272, which is the increment for the squared area (now 3457); the new ratio is 3457 to the square of 107. The count of differences of 4 is now incremented by 1, and the process repeats.



The result, for the first 100 odd primes is shown as a graph. The ratio will approach zero asymptotically as the number of primes increases. The curve goes up whenever there are long strings of differences that have previously appeared (e.g., for the primes 151, 157, 163, 167, 173, and 179). It goes down significantly with each appearance of a new difference (e.g., for the primes 113 and 127).

The appearance of the curve suggests a new problem, however. If the ratio is multiplied by the 4th root of the prime, the result seems to stabilize at around .9. In the range of the primes from the 100th to the 200th prime, it varies from .80 to .93. Does this new ratio converge, and if so, to what?


Ratio of $\frac{\text{Squared area}}{\text{Total area}}$
.45
.40
.35
.30
.25
.20
.15
.10
10
20
30
40
50
60
70
80
90
100
Number of odd primes

Problem Solution

Problem: On a 12 x 12 grid of points, form all possible sets of three points and tabulate the areas of the triangles they form.

This is the 12 x 12 version of Wendy's problem (posed in issue 45; solution in issue 47).

For the 12 x 12 array of points, there are 487,344 sets of three points. Of these, 10332 sets do not form a triangle (or, form a triangle of zero area). In the tabulation below, the column labelled N is twice the area, and the K column shows how many such triangles there are. The computation was done by Associate Editor David Babcock. □

N	K	N	K	N	K	N	K	N	K
0	10332	1	12260	2	17944	3	15436	4	19656
5	13136	6	21136	7	11508	8	18436	9	13412
10	16184	11	8452	12	18984	13	7348	14	12568
15	11540	16	13524	17	6300	18	14256	19	5832
20	12808	21	8884	22	8176	23	4984	24	12928
25	6140	26	6584	27	6748	28	8528	29	3936
30	9912	31	3632	32	7228	33	4736	34	4768
35	5172	36	7728	37	2824	38	4032	39	3336
40	6624	41	2392	42	5568	43	2192	44	3816
45	3932	46	2856	47	1840	48	4588	49	2364
50	3360	51	1928	52	2344	53	1392	54	3280
55	1860	56	3060	57	1444	58	1624	59	1032
60	2632	61	936	62	1336	63	1932	64	1728
65	916	66	1560	67	680	68	984	69	752
70	1568	71	560	72	1632	73	496	74	688
75	592	76	600	77	688	78	584	79	352
80	1028	81	644	82	416	83	264	84	384
85	268	86	312	87	224	88	520	89	176
90	656	91	160	92	176	93	136	94	168
95	136	96	132	97	96	98	120	99	284
100	232	101	64	102	64	103	48	104	64
105	44	106	56	107	40	108	32	109	32
110	176	111	16	112	12	113	16	114	8
115	16	116	8	117	12	118	8	119	8
120	4	121	44						

PC54-18

A Coding Exercise

The sequence, X, shown in the table, is formed by increments of D. The D values are successive integers, each repeated 1, 1, 2, 3, 5, 8, 13, 21,... times (namely, the number of times given by the Fibonacci sequence).

The 100th term is 770.

1) What is the 1000th term?

2) What is the Nth term?

N	X
1	1
2	2
3	4
4	7
5	10
6	14
7	18
8	22
9	27
10	32
11	37
12	42
13	47
14	53
15	59
16	65
17	71
18	77
19	83
20	89
21	95

D	
1	1
2	1
3	
3	2
4	
4	3
4	
5	
5	
5	5
5	
5	
6	
6	
6	
6	
6	8
6	
6	
6	
7	

(From a suggestion by R. W. Hamming)

PROBLEM 191

WRENCH ON PROBLEM 165

In issue 49, the old wine-and-water problem was given as a computing problem:

> One glass contains 100 cc of wine; a second glass contains 100 cc of water. One cc of wine is moved to the second glass, and then one cc of the mixture is moved back to the first glass. How does the amount of wine now in the first glass compare to the amount of water now in the second glass? How many complete transfers will it take to bring the amount down to 51 cc? To 50.5 cc? To 50.05 cc?

As seems to be customary in such cases, the problem can be demolished analytically, and John W. Wrench, Jr. has done just that:

"Let C = volume of wine = volume of water

W_n = volume of wine in first glass after n complete transfers.

Then we have the difference equation:

$$W_{n+1} = \frac{C-1}{C+1} W_n + \frac{C}{C+1}, \quad W_0 = C, \qquad \text{(A)}$$

whose solution may be written in the form:

$$W_n = \frac{C}{2}\left\{1 + \left(\frac{C-1}{C+1}\right)^n\right\}, \quad n = 0,1,2,3,\ldots. \qquad \text{(B)}$$

This implies:

$$n \ln\left(\frac{C+1}{C-1}\right) = \ln\left(\frac{C}{2W_n - C}\right). \qquad \text{(C)}$$

Since $\ln\left(\frac{C+1}{C-1}\right) = 2\left\{\frac{1}{C} + \frac{1}{3C^3} + \cdots\right\}, \quad C > 1,$

we have the approximation:

$$n = \frac{C}{2} \ln\left(\frac{C}{2W_n - C}\right) \qquad \text{(D)}$$

Thus, when $C = 100$, $W_n = 51$, we obtain $n = 195$.

From (B) we calculate $W_{195} = 51.011964$, $W_{196} = 50.991925$.

When $W_n = 50.5$ we find $n = 230$; $W_{230} = 50.502515$, $W_{231} = 50.492564$.

When $W_n = 50.05$ we find $n = 345$; $W_{345} = 50.050377$, $W_{346} = 50.049379$.

The problem, done step-by-step, is still a practical exercise for a beginner learning how to program repetitive situations. Wrench's formulas now provide an easy way to check results obtained, by computer, the hard way.

A and B acquire quartz watches. After some days, they establish the error rates of the watches as follows:

$$A: \quad y = -.000123x^2 - .284x$$

$$B: \quad y = .0000613x^2 + .179x$$

where x is in days and y is in seconds. Thus, if the watches are set correctly at startup, one year later B's watch will be 73.5 seconds fast and A's watch will be 120 seconds slow.

If the error rates of the two watches were linear (say, B's fast by <u>b</u> seconds per year and A's slow by <u>a</u> seconds per year), then the correct time could be found by the formula:

$$\frac{b}{a+b}(B - A) - B$$

But with non-linear error rates, the formula is not so simple. The Problem is: knowing the nature of the error rates, but not knowing how many days have elapsed since startup, what is the best estimate one can make of the correct time, using the readings of the two watches?

PROBLEM **192**